## REVIEW \& SUMMARY

The Biot-Savart Law The magnetic field set up by a currentcarrying conductor can be found from the Biot-Savart law. This law asserts that the contribution $d \vec{B}$ to the field produced by a current-length element $i d \vec{s}$ at a point $P$ located a distance $r$ from the current element is

$$
\begin{equation*}
d \vec{B}=\frac{\mu_{0}}{4 \pi} \frac{i d \vec{s} \times \hat{\mathrm{r}}}{r^{2}} \quad \text { (Biot-Savart law) } \tag{29-3}
\end{equation*}
$$

Here $\hat{\mathrm{r}}$ is a unit vector that points from the element toward $P$. The quantity $\mu_{0}$, called the permeability constant, has the value

$$
4 \pi \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A} \approx 1.26 \times 10^{-6} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}
$$

Magnetic Field of a Long Straight Wire For a long straight wire carrying a current $i$, the Biot-Savart law gives, for the magnitude of the magnetic field at a perpendicular distance $R$ from the wire,

$$
\begin{equation*}
B=\frac{\mu_{0} i}{2 \pi R} \quad \text { (long straight wire). } \tag{29-4}
\end{equation*}
$$

Magnetic Field of a Circular Arc The magnitude of the magnetic field at the center of a circular arc, of radius $R$ and central angle $\phi$ (in radians), carrying current $i$, is

$$
\begin{equation*}
B=\frac{\mu_{0} i \phi}{4 \pi R} \quad \text { (at center of circular arc). } \tag{29-9}
\end{equation*}
$$

Force Between Parallel Currents Parallel wires carrying currents in the same direction attract each other, whereas parallel wires carrying currents in opposite directions repel each other. The magnitude of the force on a length $L$ of either wire is

$$
\begin{equation*}
F_{b a}=i_{b} L B_{a} \sin 90^{\circ}=\frac{\mu_{0} L i_{a} i_{b}}{2 \pi d} \tag{29-13}
\end{equation*}
$$

where $d$ is the wire separation, and $i_{a}$ and $i_{b}$ are the currents in the wires.

Ampere's Law Ampere's law states that

$$
\begin{equation*}
\oint \vec{B} \cdot d \vec{s}=\mu_{0} i_{\mathrm{enc}} \quad \text { (Ampere's law). } \tag{29-14}
\end{equation*}
$$

The line integral in this equation is evaluated around a closed loop called an Amperian loop. The current $i$ on the right side is the net current encircled by the loop. For some current distributions, Eq. 29-14 is easier to use than Eq. 29-3 to calculate the magnetic field due to the currents.

Fields of a Solenoid and a Toroid Inside a long solenoid carrying current $i$, at points not near its ends, the magnitude $B$ of the magnetic field is

$$
\begin{equation*}
B=\mu_{0} \text { in } \quad \text { (ideal solenoid) }, \tag{29-23}
\end{equation*}
$$

where $n$ is the number of turns per unit length. At a point inside a toroid, the magnitude $B$ of the magnetic field is

$$
\begin{equation*}
B=\frac{\mu_{0} i N}{2 \pi} \frac{1}{r} \quad \text { (toroid) } \tag{29-24}
\end{equation*}
$$

where $r$ is the distance from the center of the toroid to the point.
Field of a Magnetic Dipole The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point $P$ located a distance $z$ along the coil's perpendicular central axis is parallel to the axis and is given by

$$
\begin{equation*}
\vec{B}(z)=\frac{\mu_{0}}{2 \pi} \frac{\vec{\mu}}{z^{3}}, \tag{29-27}
\end{equation*}
$$

where $\vec{\mu}$ is the dipole moment of the coil. This equation applies only when $z$ is much greater than the dimensions of the coil.

## UESTIONS

1 Figure 29-23 shows three circuits, each consisting of two radial lengths and two concentric circular arcs, one of radius $r$ and the other of radius $R>r$. The circuits have the same current through them and the same angle between the two radial lengths. Rank the circuits according to the magnitude of the net magnetic field at the center, greatest first.


Fig. 29-23 Question 1.
2 Figure 29-24 represents a snapshot of the velocity vectors of four electrons near a wire carrying current $i$. The four velocities
have the same magnitude; velocity $\vec{\nu}_{2}$ is directed into the page. Electrons 1 and 2 are at the same distance from the wire, as are electrons 3 and 4. Rank the electrons according to the magnitudes of the magnetic forces on them due to current $i$, greatest first.
3 Figure 29-25 shows four arrangements in which long parallel wires


Fig. 29-24 Question 2.


Fig. 29-25 Question 3.
carry equal currents directly into or out of the page at the corners of identical squares. Rank the arrangements according to the magnitude of the net magnetic field at the center of the square, greatest first.
4 Figure 29-26 shows cross sections of two long straight wires; the lefthand wire carries current $i_{1}$ directly out of the page. If the net magnetic


Fig. 29-26 Question 4. field due to the two currents is to be zero at point $P$, (a) should the direction of current $i_{2}$ in the right-hand wire be directly into or out of the page and (b) should $i_{2}$ be greater than, less than, or equal to $i_{1}$ ?
5 Figure 29-27 shows three circuits consisting of straight radial lengths and concentric circular arcs (either half- or quarter-circles of radii $r, 2 r$, and $3 r$ ). The circuits carry the same current. Rank them according to the magnitude of the magnetic field produced at the center of curvature (the dot), greatest first.


Fig. 29-27 Question 5.
6 Figure 29-28 gives, as a function of radial distance $r$, the magnitude $B$ of the magnetic field inside and outside four wires ( $a, b, c$, and $d)$, each of which carries a current that is uniformly distributed across the wire's cross section. Overlapping portions of the plots are indicated by double labels. Rank the wires according to (a) radius, (b) the magnitude of the magnetic field on the surface, and (c) the value of the current, greatest first. (d) Is the magnitude of the current density in wire $a$ greater than, less than, or equal to that in wire $c$ ?


Fig. 29-28 Question 6.
7 Figure 29-29 shows four circular Amperian loops ( $a, b, c, d$ ) concentric with a wire whose current is directed out of the page. The current is uniform across the wire's circular cross section (the shaded region). Rank the loops according to the magnitude of $\oint \vec{B} \cdot d \vec{s}$ around each, greatest first.


Fig. 29-29 Question 7.
8 Figure 29-30 shows four arrangements in which long, parallel, equally spaced wires carry equal currents directly into or out of the page. Rank the arrangements according to the magnitude of the
net force on the central wire due to the currents in the other wires, greatest first.


Fig. 29-30 Question 8.
9 Figure 29-31 shows four circular Amperian loops ( $a, b, c, d$ ) and, in cross section, four long circular conductors (the shaded regions), all of which are concentric. Three of the conductors are hollow cylinders; the central conductor is a solid cylinder. The currents in the conductors are, from smallest radius to largest radius, 4 A out of the page, 9 A into the page,


Fig. 29-31 Question 9. 5 A out of the page, and 3 A into the page. Rank the Amperian loops according to the magnitude of $\oint \vec{B} \cdot d \vec{s}$ around each, greatest first.
10 Figure 29-32 shows four identical currents $i$ and five Amperian paths ( $a$ through $e$ ) encircling them. Rank the paths according to the value of $\oint \vec{B} \cdot d \vec{s}$ taken in the directions shown, most positive first.


Fig. 29-32 Question 10.
11 Figure 29-33 shows three arrangements of three long straight wires carrying equal currents directly into or out of the page. (a) Rank the arrangements according to the magnitude of the net force on wire $A$ due to the currents in the other wires, greatest first. (b) In arrangement 3, is the angle between the net force on wire $A$ and the dashed line equal to, less than, or more than $45^{\circ}$ ?


Fig. 29-33 Question 11.


## sec. 29-2 Calculating the Magnetic Field Due to a Current

-1 A surveyor is using a magnetic compass 6.1 m below a power line in which there is a steady current of 100 A . (a) What is the magnetic field at the site of the compass due to the power line? (b) Will this field interfere seriously with the compass reading? The horizontal component of Earth's magnetic field at the site is $20 \mu \mathrm{~T}$.
-2 Figure 29-34a shows an element of length $d s=1.00 \mu \mathrm{~m}$ in a very long straight wire carrying current. The current in that element sets up a differential magnetic field $d \vec{B}$ at points in the surrounding space. Figure 29-34b gives the magnitude $d B$ of the field for points 2.5 cm from the element, as a function of angle $\theta$ between the wire and a straight line to the point. The vertical scale is set by $d B_{s}=60.0 \mathrm{pT}$. What is the magnitude of the magnetic field set up by the entire wire at perpendicular distance 2.5 cm

(a)

(b)

Fig. 29-34 Problem 2.
set up at point $P$ is due to contributions from all the identical currentlength elements $i d \vec{s}$ along the wire. What is the distance $s$ to the element making (a) the greatest contribution to field $\vec{B}$ and (b) $10.0 \%$ of the greatest contribution?
$\cdot 7$ In Fig. 29-38, two circular arcs have radii $a=13.5 \mathrm{~cm}$ and $b=10.7 \mathrm{~cm}$, subtend angle $\theta=74.0^{\circ}$, carry current $i$ $=0.411 \mathrm{~A}$, and share the same center of curvature $P$. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at $P$ ?
-8 In Fig. 29-39, two semicircular arcs have radii $R_{2}=7.80 \mathrm{~cm}$ and $R_{1}=3.15$ cm , carry current $i=0.281 \mathrm{~A}$, and share the same center of curvature $C$. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at $C$ ?
-9 SSM Two long straight wires are parallel and 8.0 cm apart. They are to carry equal currents such that the magnetic field at a point halfway between them has magnitude $300 \mu \mathrm{~T}$. (a) Should the currents be in the same or opposite directions? (b) How much current is needed?
-10 In Fig. 29-40, a wire forms a semicircle of radius $R=9.26 \mathrm{~cm}$ and two (radial) straight segments each of length $L=13.1 \mathrm{~cm}$. The wire carries current $i=34.8 \mathrm{~mA}$. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the semicircle's center of curvature $C$ ?
-11 In Fig. 29-41, two long straight wires are perpendicular to the page and separated by distance $d_{1}=0.75$ cm . Wire 1 carries 6.5 A into the page. What are the (a) magnitude and (b) direction (into or out of the page) of the current in wire 2 if the net magnetic field due to the two currents is zero at point $P$ located at distance $d_{2}=1.50 \mathrm{~cm}$ from wire 2?
-12 In Fig. 29-42, two long straight wires at separation $d=16.0 \mathrm{~cm}$ carry currents $i_{1}=3.61 \mathrm{~mA}$ and $i_{2}=3.00 i_{1}$ out of the page. (a) Where on the $x$ axis is the net magnetic field equal to zero? (b) If the two currents are doubled, is the zero-field point shifted toward wire 1 , shifted toward wire 2 , or unchanged?


Fig. 29-38 Problem 7.


Fig. 29-39 Problem 8.


Fig. 29-40 Problem 10.


Fig. 29-41 Problem 11.


Fig. 29-42 Problem 12.
-3 SSM At a certain location in the Philippines, Earth's magnetic field of $39 \mu \mathrm{~T}$ is horizontal and directed due north. Suppose the net field is zero exactly 8.0 cm above a long, straight, horizontal wire that carries a constant current. What are the (a) magnitude and (b) direction of the current?
-4 A straight conductor carrying current $i=5.0 \mathrm{~A}$ splits into identical semicircular arcs as shown in Fig. 29-35. What is the magnetic field at the center $C$ of the resulting circular loop?
-5 In Fig. 29-36, a current $i=10 \mathrm{~A}$ is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius $R=5.0 \mathrm{~mm}$. Point $b$ is midway between the straight sections and so distant from the semicircle that each straight section can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of $\vec{B}$ at $a$ and the (c) magnitude and (d) direction of $\vec{B}$ at $b$ ?
${ }^{-} 6$ In Fig. 29-37, point $P$ is at perpendicular distance $R=2.00 \mathrm{~cm}$ from a very long straight wire carrying a current. The magnetic field $\vec{B}$


Fig. 29-35 Problem 4.


Fig. 29-36 Problem 5.


Fig. 29-37 Problem 6.
$\bullet 13$ In Fig. 29-43, point $P_{1}$ is at distance $R=13.1 \mathrm{~cm}$ on the perpendicular bisector of a straight wire of length $L=18.0 \mathrm{~cm}$ carrying current $i=58.2 \mathrm{~mA}$. (Note that the wire is not long.) What is the magnitude of the magnetic field at $P_{1}$ due to $i$ ?
$\bullet 14$ Equation 29-4 gives the magnitude $B$ of the magnetic field set up by a current in an infinitely long straight wire, at a point $P$ at perpendicular distance $R$ from the wire. Suppose that point $P$ is actually at perpendicular distance $R$ from the midpoint of a wire with a finite length $L$. Using Eq. 29-4 to calculate $B$ then results in a certain percentage error. What value must the ratio $L / R$ exceed if the percentage error is to be less than $1.00 \%$ ? That is, what $L / R$ gives

$$
\frac{(B \text { from Eq. } 29-4)-(B \text { actual })}{(B \text { actual })}(100 \%)=1.00 \% ?
$$

-•15 Figure 29-44 shows two current segments. The lower segment carries a current of $i_{1}=0.40 \mathrm{~A}$ and includes a semicircular arc with radius 5.0 cm , angle $180^{\circ}$, and center point $P$. The upper segment carries current $i_{2}=2 i_{1}$ and includes a circular arc with radius 4.0 cm , angle


Fig. 29-44 Problem 15. $120^{\circ}$, and the same center point $P$. What are the (a) magnitude and (b) direction of the net magnetic field $\vec{B}$ at $P$ for the indicated current directions? What are the (c) magnitude and (d) direction of $\vec{B}$ if $i_{1}$ is reversed?
$\bullet 16$ In Fig. 29-45, two concentric circular loops of wire carrying current in the same direction lie in the same plane. Loop 1 has radius 1.50 cm and carries 4.00 mA . Loop 2 has radius 2.50 cm and carries 6.00


Fig. 29-45 Problem 16. mA . Loop 2 is to be rotated about a diameter while the net magnetic field $\vec{B}$ set up by the two loops at their common center is measured. Through what angle must loop 2 be rotated so that the magnitude of that net field is 100 nT ?
$\bullet 17$ SSM In Fig. 29-43, point $P_{2}$ is at perpendicular distance $R=$ 25.1 cm from one end of a straight wire of length $L=13.6 \mathrm{~cm}$ carrying current $i=0.693 \mathrm{~A}$. (Note that the wire is not long.) What is the magnitude of the magnetic field at $P_{2}$ ?
$\bullet 18$ A current is set up in a wire loop consisting of a semicircle of radius 4.00 cm , a smaller concentric semicircle, and two radial straight lengths, all in the same plane. Figure $29-46 a$ shows the arrangement but is

(a)

Fig. 29-46 Problem 18. not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is $47.25 \mu \mathrm{~T}$. The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29-46b). The magnetic field produced at the (same) center of curvature now has magnitude $15.75 \mu \mathrm{~T}$, and its direction is reversed. What is the radius of the smaller semicircle?
-1 19 One long wire lies along an $x$ axis and carries a current of 30 A in the positive $x$ direction. A second long wire is perpendicular to the $x y$ plane, passes through the point $(0,4.0 \mathrm{~m}, 0)$, and carries a current of 40 A in the positive $z$ direction. What is the magnitude of the resulting magnetic field at the point $(0,2.0 \mathrm{~m}, 0)$ ?
-20 In Fig. 29-47, part of a long insulated wire carrying current $i=5.78 \mathrm{~mA}$ is bent into a circular section of radius $R=1.89 \mathrm{~cm}$. In unit-vector notation, what is the magnetic field at the center of curvature $C$ if the circular section (a) lies in the plane of the page as shown and (b) is perpendicular to the plane of the page after being rotated $90^{\circ}$ counterclockwise as indicated?
-21 Figure 29-48 shows two very long straight wires (in cross section) that each carry a current of 4.00 A directly out of the page. Distance $d_{1}=6.00 \mathrm{~m}$ and distance $d_{2}=4.00 \mathrm{~m}$. What is the magnitude of the net magnetic field at point $P$, which lies on a perpendicular bisector to the wires?


Fig. 29-47 Problem 20.


Fig. 29-48 Problem 21.
-०22 Figure 29-49a shows, in cross section, two long, parallel wires carrying current and separated by distance $L$. The ratio $i_{1} / i_{2}$ of their currents is 4.00 ; the directions of the currents are not indicated. Figure 29-49b shows the $y$ component $B_{y}$ of their net magnetic field along the $x$ axis to the right of wire 2. The vertical scale is set by $B_{y s}=4.0 \mathrm{nT}$, and the horizontal scale is set by $x_{s}=20.0 \mathrm{~cm}$. (a) At what value of $x>0$ is $B_{y}$ maximum? (b) If $i_{2}=3 \mathrm{~mA}$, what is the value of that maximum? What is the direction (into or out of the page) of (c) $i_{1}$ and (d) $i_{2}$ ?


Fig. 29-49 Problem 22.
-23 ILW Figure 29-50 shows a snapshot of a proton moving at velocity $\vec{v}=(-200 \mathrm{~m} / \mathrm{s}) \hat{\mathrm{j}}$ toward a long straight wire with current $i=$ 350 mA . At the instant shown, the proton's distance from the wire is


Fig. 29-50 Problem 23. $d=2.89 \mathrm{~cm}$. In unit-vector notation, what is the magnetic force on the proton due to the current? -24 Figure 29-51 shows, in cross section, four thin wires that are parallel, straight, and very long. They carry identical currents in the directions indicated. Initially all four wires are at distance $d=$ 15.0 cm from the origin of the coordinate system, where they cre-
ate a net magnetic field $\vec{B}$. (a) To what value of $x$ must you move wire 1 along the $x$ axis in order to rotate $\vec{B}$ counterclockwise by $30^{\circ}$ ? (b) With wire 1 in that new position, to what value of $x$ must you move wire 3 along the $x$ axis to rotate $\vec{B}$ by $30^{\circ}$ back to its initial orientation?
$\bullet 25$ SSM A wire with current $i=3.00 \mathrm{~A}$ is shown in Fig. 29-52. Two semi-infinite straight sections, both tangent to the same circle, are connected by a circular arc that has a central angle $\theta$ and runs along the circumference of the circle. The arc and the two straight sections all lie in the same plane. If $B=0$ at the circle's center, what is $\theta$ ?
-26 In Fig. 29-53a, wire 1 consists of a circular arc and two radial lengths; it


Fig. 29-51
Problem 24.


Fig. 29-52
Problem 25. carries current $i_{1}=0.50 \mathrm{~A}$ in the direction indicated. Wire 2 , shown in cross section, is long, straight, and perpendicular to the plane of the figure. Its distance from the center of the arc is equal to the radius $R$ of the arc, and it carries a current $i_{2}$ that can be varied. The two currents set up a net magnetic field $\vec{B}$ at the center of the arc. Figure $29-53 b$ gives the square of the field's magnitude $B^{2}$ plotted versus the square of the current $i_{2}^{2}$. The vertical scale is set by $B_{s}^{2}=10.0 \times 10^{-10} \mathrm{~T}^{2}$. What angle is subtended by the arc?

(a)

(b)

Fig. 29-53 Problem 26.
-•27 In Fig. 29-54, two long straight wires (shown in cross section) carry currents $i_{1}=30.0 \mathrm{~mA}$ and $i_{2}=40.0$ mA directly out of the page. They are equal distances from the origin, where they set up a magnetic field $\vec{B}$. To what value must current $i_{1}$ be changed in order to rotate $\vec{B} 20.0^{\circ}$ clockwise?


Fig. 29-54 Problem 27.
-28 5igure 29-55a shows two
wires, each carrying a current. Wire 1 consists of a circular arc of radius $R$ and two radial lengths; it carries current $i_{1}=2.0 \mathrm{~A}$ in the direction indicated. Wire 2 is long and straight; it carries a current $i_{2}$ that can be varied; and it is at distance $R / 2$ from the center of the arc. The net magnetic field $\vec{B}$ due to the two currents is measured at
the center of curvature of the arc. Figure $29-55 b$ is a plot of the component of $\vec{B}$ in the direction perpendicular to the figure as a function of current $i_{2}$. The horizontal scale is set by $i_{2 s}=1.00 \mathrm{~A}$. What is the angle subtended by the arc?


Fig. 29-55 Problem 28.
-•29 SSM In Fig. 29-56, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a=20 \mathrm{~cm}$. The currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3 , and each wire carries 20 A . In unit-vector notation, what is the net magnetic field at the square's center?
-0030 Two long straight thin wires with current lie against an equally long plastic cylinder, at ra-


Fig. 29-56
Problems 29, 37 , and 40. dius $R=20.0 \mathrm{~cm}$ from the cylinder's central axis. Figure 29-57a shows, in cross section, the cylinder and wire 1 but not wire 2 . With wire 2 fixed in place, wire 1 is moved around the cylinder, from angle $\theta_{1}=0^{\circ}$ to angle $\theta_{1}=180^{\circ}$, through the first and second quadrants of the $x y$ coordinate system. The net magnetic field

(a)

(b)

(c)

Fig. 29-57 Problem 30.
$\vec{B}$ at the center of the cylinder is measured as a function of $\theta_{1}$. Figure 29-57b gives the $x$ component $B_{x}$ of that field as a function of $\theta_{1}$ (the vertical scale is set by $B_{x s}=6.0 \mu \mathrm{~T}$ ), and Fig. 29-57c gives the $y$ component $B_{y}$ (the vertical scale is set by $B_{y s}=4.0$ $\mu \mathrm{T}$ ). (a) At what angle $\theta_{2}$ is wire 2 located? What are the (b) size and (c) direction (into or out of the page) of the current in wire 1 and the (d) size and (e) direction of the current in wire 2?
-0031 In Fig. 29-58, length $a$ is 4.7 cm (short) and current $i$ is 13 A . What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at point $P$ ?

0032 The current-carrying wire


Fig. 29-58 Problem 31. loop in Fig. 29-59a lies all in one plane and consists of a semicircle of radius 10.0 cm , a smaller semicircle with the same center, and two radial lengths. The smaller semicircle is rotated out of that plane by angle $\theta$, until it is perpendicular to the plane (Fig. 29-59b). Figure 29-59c gives the magnitude of the net magnetic field at the center of curvature versus angle $\theta$. The vertical scale is set by $B_{a}=10.0 \mu \mathrm{~T}$ and $B_{b}=12.0 \mu \mathrm{~T}$. What is the radius of the smaller semicircle?


(c)

Fig. 29-59 Problem 32.
00033 SSM ILW Figure 29-60 shows a cross section of a long thin ribbon of width $w=4.91 \mathrm{~cm}$ that is carrying a uniformly distributed total current $i=4.61 \mu \mathrm{~A}$ into the page. In unit-vector notation, what is the magnetic field $\vec{B}$ at a point $P$ in the plane of the ribbon at a distance $d=$ 2.16 cm from its edge? (Hint: Imagine the ribbon as being constructed from many long, thin, parallel wires.)
00034 Figure 29-61 shows, in cross section, two long straight wires held against a plastic cylinder of radius 20.0 cm . Wire 1 carries current $i_{1}=$ 60.0 mA out of the page and is fixed in place at the left side of the cylinder. Wire 2 carries current $i_{2}=40.0$


Fig. 29-60 Problem 33.


Fig. 29-61 Problem 34.
mA out of the page and can be moved around the cylinder. At what (positive) angle $\theta_{2}$ should wire 2 be positioned such that, at the origin, the net magnetic field due to the two currents has magnitude 80.0 nT ?

## sec. 29-3 Force Between Two Parallel Currents

-35 SSM Figure 29-62 shows wire 1 in cross section; the wire is long and straight, carries a current of 4.00 mA out of the page, and is at distance $d_{1}=2.40 \mathrm{~cm}$ from a surface. Wire 2 , which is parallel to wire 1 and also long, is at horizontal distance $d_{2}=5.00 \mathrm{~cm}$ from


Fig. 29-62 Problem 35. wire 1 and carries a current of 6.80 mA into the page. What is the $x$ component of the magnetic force per unit length on wire 2 due to wire 1 ?
-036 In Fig. 29-63, five long parallel wires in an $x y$ plane are separated by distance $d=8.00 \mathrm{~cm}$, have lengths of 10.0 m , and carry identical currents of 3.00 A out of the page. Each wire experiences a magnetic force due to the other wires. In unit-vector


Fig. 29-63 Problems 36 and 39 . notation, what is the net magnetic force on (a) wire 1, (b) wire 2, (c) wire 3, (d) wire 4, and (e) wire 5?
-037 In Fig. 29-56, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a=13.5 \mathrm{~cm}$. Each wire carries 7.50 A , and the currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3 . In unitvector notation, what is the net magnetic force per meter of wire length on wire 4 ?
-038 Figure 29-64a shows, in cross section, three current-carrying wires that are long, straight, and parallel to one another. Wires 1 and 2 are fixed in place on an $x$ axis, with separation $d$. Wire 1 has a current of 0.750 A , but the direction of the current is not given. Wire 3, with a current of 0.250 A out of the page, can be moved along the $x$ axis to the right of wire 2 . As wire 3 is moved, the magnitude of the net magnetic force $\vec{F}_{2}$ on wire 2 due to the currents in wires 1 and 3 changes. The $x$ component of that force is $F_{2 x}$ and the value per unit length of wire 2 is $F_{2 x} / L_{2}$. Figure 29-64b gives $F_{2 x} / L_{2}$ versus the position $x$ of wire 3 . The plot has an asymptote $F_{2 x} / L_{2}=$ $-0.627 \mu \mathrm{~N} / \mathrm{m}$ as $x \rightarrow \infty$. The horizontal scale is set by $x_{s}=12.0 \mathrm{~cm}$. What are the (a) size and (b) direction (into or out of the page) of the current in wire 2 ?


Fig. 29-64 Problem 38.
-•39
In Fig. 29-63, five long parallel wires in an $x y$ plane are separated by distance $d=50.0 \mathrm{~cm}$. The currents into the page are
$i_{1}=2.00 \mathrm{~A}, i_{3}=0.250 \mathrm{~A}, i_{4}=4.00 \mathrm{~A}$, and $i_{5}=2.00 \mathrm{~A}$; the current out of the page is $i_{2}=4.00 \mathrm{~A}$. What is the magnitude of the net force per unit length acting on wire 3 due to the currents in the other wires?
$\bullet 40$ In Fig. 29-56, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length $a=8.50 \mathrm{~cm}$. Each wire carries 15.0 A , and all the currents are out of the page. In unit-vector notation, what is the net magnetic force per meter of wire length on wire 1 ?
$\bullet \bullet 41$ ILW In Fig. 29-65, a long straight wire carries a current $i_{1}=$ 30.0 A and a rectangular loop carries current $i_{2}=20.0 \mathrm{~A}$. Take $a=$ $1.00 \mathrm{~cm}, b=8.00 \mathrm{~cm}$, and $L=30.0$ cm . In unit-vector notation, what is the net force on the loop due to $i_{1}$ ?


Fig. 29-65 Problem 41.

## sec. 29-4 Ampere's Law

-42 In a particular region there is a uniform current density of $15 \mathrm{~A} / \mathrm{m}^{2}$ in the positive $z$ direction. What is the value of $\oint \vec{B} \cdot d \vec{s}$ when that line integral is calculated along the three straight-line segments from $(x, y, z)$ coordinates ( $4 d, 0$, $0)$ to $(4 d, 3 d, 0)$ to $(0,0,0)$ to $(4 d, 0,0)$, where $d=20 \mathrm{~cm}$ ?
-43 Figure 29-66 shows a cross section across a diameter of a long cylindrical conductor of radius $a=2.00 \mathrm{~cm}$ carrying uniform current 170 A . What is the magnitude of the current's magnetic field at radial distance (a) 0 , (b) 1.00 cm , (c) 2.00 cm (wire's surface), and


Fig. 29-66
Problem 43. (d) 4.00 cm ?
-44 Figure 29-67 shows two closed paths wrapped around two conducting loops carrying currents $i_{1}=5.0 \mathrm{~A}$ and $i_{2}=3.0 \mathrm{~A}$. What is the value of the integral $\oint \vec{B} \cdot d \vec{s}$ for (a) path 1 and (b) path 2?
.45 ssm Each of the eight conductors in Fig. 29-68 carries 2.0 A of current into or out of the page. Two paths are indicated for the line integral $\oint \vec{B} \cdot d \vec{s}$. What is the value of the integral for (a) path 1 and (b) path 2 ?
-46 Eight wires cut the page perpendicularly at the points shown in Fig. 29-69. A wire labeled with the integer $k(k=1,2, \ldots, 8)$ carries the current $k i$, where $i=4.50 \mathrm{~mA}$. For those wires with odd $k$, the current is out of the page; for those with even $k$, it is into the page. Evaluate $\oint \vec{B} \cdot d \vec{s}$ along the closed path in the direction shown.
-047 ILw The current density $\vec{J}$ inside a long, solid, cylindrical wire


Fig. 29-67 Problem 44.


Fig. 29-68 Problem 45.


Fig. 29-69 Problem 46.
of radius $a=3.1 \mathrm{~mm}$ is in the direction of the central axis, and its magnitude varies linearly with radial distance $r$ from the axis according to $J=J_{0} r / a$, where $J_{0}=310 \mathrm{~A} / \mathrm{m}^{2}$. Find the magnitude of the magnetic field at (a) $r=0$, (b) $r=a / 2$, and (c) $r=a$.
-•48 In Fig. 29-70, a long circular pipe with outside radius $R=2.6 \mathrm{~cm}$ carries a (uniformly distributed) current $i=$ 8.00 mA into the page. A wire runs parallel to the pipe at a distance of 3.00 R from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point $P$ has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.


Fig. 29-70
Problem 48.

## sec. 29-5 Solenoids and Toroids

-49 A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of 0.800 A . (It is made up of a square solenoid -instead of a round one as in Fig. 29-16 - bent into a doughnut shape.) What is the magnetic field inside the toroid at (a) the inner radius and (b) the outer radius?
-50 A solenoid that is 95.0 cm long has a radius of 2.00 cm and a winding of 1200 turns; it carries a current of 3.60 A . Calculate the magnitude of the magnetic field inside the solenoid.
-51 A 200-turn solenoid having a length of 25 cm and a diameter of 10 cm carries a current of 0.29 A . Calculate the magnitude of the magnetic field $\vec{B}$ inside the solenoid.
-52 A solenoid 1.30 m long and 2.60 cm in diameter carries a current of 18.0 A . The magnetic field inside the solenoid is 23.0 mT . Find the length of the wire forming the solenoid.
-•53 A long solenoid has 100 turns/cm and carries current i. An electron moves within the solenoid in a circle of radius 2.30 cm perpendicular to the solenoid axis. The speed of the electron is $0.0460 c$ ( $c=$ speed of light). Find the current $i$ in the solenoid.
$\bullet 54$ An electron is shot into one end of a solenoid. As it enters the uniform magnetic field within the solenoid, its speed is 800 $\mathrm{m} / \mathrm{s}$ and its velocity vector makes an angle of $30^{\circ}$ with the central axis of the solenoid. The solenoid carries 4.0 A and has 8000 turns along its length. How many revolutions does the electron make along its helical path within the solenoid by the time it emerges from the solenoid's opposite end? (In a real solenoid, where the field is not uniform at the two ends, the number of revolutions would be slightly less than the answer here.)
-055 SSM ILW www A long solenoid with 10.0 turns $/ \mathrm{cm}$ and a radius of 7.00 cm carries a current of 20.0 mA . A current of 6.00 A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at $45.0^{\circ}$ to the axial direction? (b) What is the magnitude of the magnetic field there?

## sec. 29-6 A Current-Carrying Coil as a Magnetic Dipole

-56 Figure 29-71 shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of 200 turns and radius
$R=25.0 \mathrm{~cm}$, separated by a distance $s=R$. The two coils carry equal currents $i=12.2 \mathrm{~mA}$ in the same direction. Find the magnitude of the net magnetic field at $P$, midway between the coils.


Fig. 29-71 Problems 56 and 90.
-57 SSM A student makes a short electromagnet by winding 300 turns of wire around a wooden cylinder of diameter $d=5.0$ cm . The coil is connected to a battery producing a current of 4.0 A in the wire. (a) What is the magnitude of the magnetic dipole moment of this device? (b) At what axial distance $z \gg d$ will the magnetic field have the magnitude $5.0 \mu \mathrm{~T}$ (approximately one-tenth that of Earth's magnetic field)?
-58 Figure 29-72a shows a length of wire carrying a current $i$ and bent into a circular coil of one turn. In Fig. $29-72 b$ the same length of wire has been bent to give a coil of two turns, each of half the original radius. (a) If $B_{a}$ and $B_{b}$ are the magnitudes of the magnetic fields at the centers of the two coils, what is the ratio $B_{b} / B_{a}$ ? (b) What is the ratio $\mu_{b} / \mu_{a}$ of the dipole moment magnitudes of the coils?
-59 SSM What is the magnitude of the magnetic dipole moment $\vec{\mu}$ of the solenoid described in Problem 51?
©60 In Fig. 29-73a, two circular loops, with different currents but the same radius of 4.0 cm , are centered on a $y$ axis. They are initially separated by distance $L=3.0 \mathrm{~cm}$, with loop 2 positioned at the origin of the axis. The currents in the two loops produce a net magnetic field at the origin, with $y$ component $B_{y}$. That component is to be measured as loop 2 is gradually moved in the positive direction of the $y$ axis. Figure 29-73b gives $B_{y}$ as a function of the position $y$ of loop 2. The curve approaches an asymptote of $B_{y}=7.20 \mu \mathrm{~T}$ as $y \rightarrow \infty$. The horizontal scale is set by $y_{s}=10.0 \mathrm{~cm}$. What are (a) current $i_{1}$ in loop 1 and (b) current $i_{2}$ in loop 2?


Fig. 29-73 Problem 60.
-•61 A circular loop of radius 12 cm carries a current of 15 A . A flat coil of radius 0.82 cm , having 50 turns and a current of 1.3 A , is
concentric with the loop. The plane of the loop is perpendicular to the plane of the coil. Assume the loop's magnetic field is uniform across the coil. What is the magnitude of (a) the magnetic field produced by the loop at its center and (b) the torque on the coil due to the loop?
-•62 In Fig. 29-74, current $i=$ 56.2 mA is set up in a loop having two radial lengths and two semicircles of radii $a=5.72 \mathrm{~cm}$ and $b=$ 9.36 cm with a common center $P$. What are the (a) magnitude and (b) direction (into or out of the page) of the magnetic field at $P$ and the (c) magnitude and (d) direction of the loop's magnetic dipole moment?
-•63 In Fig. 29-75, a conductor carries 6.0 A along the closed path abcdefgha running along 8 of the 12 edges of a cube of edge length 10 cm . (a) Taking the path to be a combination of three square current loops (bcfgb, abgha, and $c d e f c$ ), find the net magnetic moment of the path in unit-vector notation. (b) What is the magnitude of the net magnetic field at the $x y z$ coordinates of $(0,5.0 \mathrm{~m}, 0)$ ?

## Additional Problems

64 In Fig. 29-76, a closed loop carries current $i=200 \mathrm{~mA}$. The loop consists of two radial straight wires and two concentric circular arcs of radii 2.00 m and 4.00 m . The angle $\theta$ is $\pi / 4 \mathrm{rad}$. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at the center of curvature $P$ ?
65 A cylindrical cable of radius


Fig. 29-74 Problem 62.


Fig. 29-75 Problem 63.


Fig. 29-76 Problem 64. 8.00 mm carries a current of 25.0 A , uniformly spread over its cross-sectional area. At what distance from the center of the wire is there a point within the wire where the magnetic field magnitude is 0.100 mT ?
66 Two long wires lie in an $x y$ plane, and each carries a current in the positive direction of the $x$ axis. Wire 1 is at $y=10.0 \mathrm{~cm}$ and carries 6.00 A ; wire 2 is at $y=5.00 \mathrm{~cm}$ and carries 10.0 A . (a) In unitvector notation, what is the net magnetic field $\vec{B}$ at the origin? (b) At what value of $y$ does $\vec{B}=0$ ? (c) If the current in wire 1 is reversed, at what value of $y$ does $\vec{B}=0$ ?
67 Two wires, both of length $L$, are formed into a circle and a square, and each carries current $i$. Show that the square produces a greater magnetic field at its center than the circle produces at its center.

68 A long straight wire carries a current of 50 A . An electron, traveling at $1.0 \times 10^{7} \mathrm{~m} / \mathrm{s}$, is 5.0 cm from the wire. What is the magnitude of the magnetic force on the electron if the electron velocity is directed (a) toward the wire, (b) parallel to the wire in the direction of the current, and (c) perpendicular to the two directions defined by (a) and (b)?

69 Three long wires are parallel to a $z$ axis, and each carries a current of 10 A in the positive $z$ direction. Their points of intersection with the $x y$ plane form an equilateral triangle with sides of 50 cm , as shown in Fig. 29-77. A fourth wire (wire $b$ ) passes through the midpoint of the base of the triangle and is parallel to the other three wires. If the net magnetic force on wire $a$ is zero, what are the (a) size and (b) direction ( $+z$ or $-z$ ) of the current in wire $b$ ?

70 Figure 29-78 shows a closed loop with current $i=2.00 \mathrm{~A}$. The loop consists of a half-circle of radius 4.00 m , two quarter-circles each of radius 2.00 m , and three radial straight wires. What is the magnitude of the net magnetic field at the common center of the circular sections?
71 A 10-gauge bare copper wire ( 2.6 mm in diameter) can carry a current of 50 A without overheating. For this current, what is the magnitude of the magnetic field at the surface of the wire?

72 A long vertical wire carries an unknown current. Coaxial with the wire is a long, thin, cylindrical conducting surface that carries a current of 30 mA upward. The cylindrical surface has a radius of 3.0 mm . If the magnitude of the magnetic field at a point 5.0 mm from the wire is $1.0 \mu \mathrm{~T}$, what are the (a) size and (b) direction of the current in the wire?

73 Figure 29-79 shows a cross section of a long cylindrical conductor of radius $a=4.00$ cm containing a long cylindrical hole of radius $b=1.50 \mathrm{~cm}$. The central axes of the cylinder and hole are parallel and are distance $d=$ 2.00 cm apart; current $i=5.25 \mathrm{~A}$ is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? (b) Discuss the two special cases $b=0$ and $d=0$.


Fig. 29-79
Problem 73.

74 The magnitude of the magnetic field 88.0 cm from the axis of a long straight wire is $7.30 \mu \mathrm{~T}$. What is the current in the wire?
75 SSM Figure 29-80 shows a wire segment of length $\Delta s=3.0 \mathrm{~cm}$, centered at the origin, carrying current $i$ $=2.0 \mathrm{~A}$ in the positive $y$ direction (as part of some complete circuit). To calculate the magnitude of the magnetic field $\vec{B}$ produced by the segment at a point several meters from the origin, we can use $B=\left(\mu_{0} / 4 \pi\right) i \Delta s(\sin \theta) / r^{2}$ as the Biot-Savart law. This is


Fig. 29-80 Problem 75. because $r$ and $\theta$ are essentially constant over the segment. Calculate $\vec{B}$ (in unit-vector notation) at the ( $x$, $y, z)$ coordinates (a) $(0,0,5.0 \mathrm{~m})$, (b) $(0,6.0 \mathrm{~m}, 0)$, (c) $(7.0 \mathrm{~m}, 7.0 \mathrm{~m}, 0)$, and (d) $(-3.0 \mathrm{~m},-4.0 \mathrm{~m}, 0)$.
76 Figure $29-81$ shows, in cross section, two long parallel wires spaced by distance $d=10.0 \mathrm{~cm}$; each carries 100 A , out of the

Fig. 29-77 Problem 69.


Fig. 29-78 Problem 70.
page in wire 1 . Point $P$ is on a perpendicular bisector of the line connecting the wires. In unit-vector notation, what is the net magnetic field at $P$ if the current in wire 2 is (a) out of the page and (b) into the page?
77 In Fig. 29-82, two infinitely long


Fig. 29-81 Problem 76. wires carry equal currents $i$. Each follows a $90^{\circ}$ arc on the circumference of the same circle of radius $R$. Show that the magnetic field $\vec{B}$ at the center of the circle is the same as the field $\vec{B}$ a distance $R$ below an infinite straight wire carrying a current $i$ to the left.

78 A long wire carrying 100 A is perpendicular to the magnetic field lines of a uniform magnetic field of magnitude 5.0 mT . At what distance from the wire is


Fig. 29-82
Problem 77. the net magnetic field equal to zero?
79 A long, hollow, cylindrical conductor (with inner radius 2.0 mm and outer radius 4.0 mm ) carries a current of 24 A distributed uniformly across its cross section. A long thin wire that is coaxial with the cylinder carries a current of 24 A in the opposite direction. What is the magnitude of the magnetic field (a) 1.0 mm , (b) 3.0 mm , and (c) 5.0 mm from the central axis of the wire and cylinder?
80 A long wire is known to have a radius greater than 4.0 mm and to carry a current that is uniformly distributed over its cross section. The magnitude of the magnetic field due to that current is 0.28 mT at a point 4.0 mm from the axis of the wire, and 0.20 mT at a point 10 mm from the axis of the wire. What is the radius of the wire?

81 SSM Figure 29-83 shows a cross section of an infinite conducting sheet carrying a current per unit $x$-length of $\lambda$; the current emerges perpendicularly out of the page. (a) Use the Biot-Savart law and symmetry to show that for all


Fig. 29-83 Problem 81. points $P$ above the sheet and all points $P^{\prime}$ below it, the magnetic field $\vec{B}$ is parallel to the sheet and directed as shown. (b) Use Ampere's law to prove that $B=\frac{1}{2} \mu_{0} \lambda$ at all points $P$ and $P^{\prime}$.
82 Figure 29-84 shows, in cross section, two long parallel wires that are separated by distance $d=18.6 \mathrm{~cm}$. Each carries 4.23 A , out of the page in wire 1 and into the page in wire 2 . In unit-vector notation, what is the net magnetic field at point $P$ at distance $R=$ 34.2 cm , due to the two currents?


Fig. 29-84 Problem 82.

83 SSM In unit-vector notation, what is the magnetic field at point $P$ in Fig. 29-85 if $i=10 \mathrm{~A}$ and $a=$ 8.0 cm ? (Note that the wires are not long.)
84 Three long wires all lie in an $x y$ plane parallel to the $x$ axis. They are spaced equally, 10 cm apart. The two outer wires each carry a current of 5.0 A in the positive $x$ direction. What is the magnitude of the force


Fig. 29-85 Problem 83. on a 3.0 m section of either of the outer wires if the current in the center wire is 3.2 A (a) in the positive $x$ direction and (b) in the negative $x$ direction?
85 SSM Figure 29-86 shows a cross section of a hollow cylindrical conductor of radii $a$ and $b$, carrying a uniformly distributed current $i$. (a) Show that the magnetic field magnitude $B(r)$ for the radial distance $r$ in the range $b<r<a$ is given by

$$
B=\frac{\mu_{0} i}{2 \pi\left(a^{2}-b^{2}\right)} \frac{r^{2}-b^{2}}{r} .
$$

(b) Show that when $r=a$, this equation gives the magnetic field magnitude $B$ at the surface of a long straight wire carrying current $i$; when $r=b$, it gives zero magnetic field; and when $b=0$, it gives the magnetic field inside a solid conductor of radius $a$ carrying current $i$. (c) Assume that $a=2.0 \mathrm{~cm}, b=1.8$ cm , and $i=100 \mathrm{~A}$, and then plot $B(r)$ for the range $0<r<6 \mathrm{~cm}$.


Fig. 29-86 Problem 85.

86 Show that the magnitude of the magnetic field produced at the center of a rectangular loop of wire of length $L$ and width $W$, carrying a current $i$, is

$$
B=\frac{2 \mu_{0} i}{\pi} \frac{\left(L^{2}+W^{2}\right)^{1 / 2}}{L W} .
$$

87 Figure 29-87 shows a cross section of a long conducting coaxial cable and gives its radii ( $a, b, c$ ). Equal but opposite currents $i$ are uniformly distributed in the two conductors. Derive expressions for $B(r)$ with radial distance $r$ in the ranges (a) $r<c$, (b) $c<r<$ $b$, (c) $b<r<a$, and (d) $r>a$. (e) Test these expressions for all the special cases that occur to you. (f) Assume that $a=2.0 \mathrm{~cm}, b=1.8$ $\mathrm{cm}, c=0.40 \mathrm{~cm}$, and $i=120 \mathrm{~A}$ and plot the


Fig. 29-87 Problem 87. function $B(r)$ over the range $0<r<3 \mathrm{~cm}$.

88 Figure 29-88 is an idealized schematic drawing of a rail gun. Projectile $P$ sits between two wide rails of circular cross section; a


Fig. 29-88 Problem 88.
source of current sends current through the rails and through the (conducting) projectile (a fuse is not used). (a) Let $w$ be the distance between the rails, $R$ the radius of each rail, and $i$ the current. Show that the force on the projectile is directed to the right along the rails and is given approximately by

$$
F=\frac{i^{2} \mu_{0}}{2 \pi} \ln \frac{w+R}{R} .
$$

(b) If the projectile starts from the left end of the rails at rest, find the speed $v$ at which it is expelled at the right. Assume that $i=450 \mathrm{kA}$, $w=12 \mathrm{~mm}, R=6.7 \mathrm{~cm}, L=4.0 \mathrm{~m}$, and the projectile mass is 10 g .
89 A square loop of wire of edge length $a$ carries current $i$. Show that, at the center of the loop, the magnitude of the magnetic field produced by the current is

$$
B=\frac{2 \sqrt{2} \mu_{0} i}{\pi a} .
$$

90 In Fig. 29-71, an arrangement known as Helmholtz coils consists of two circular coaxial coils, each of $N$ turns and radius $R$, separated by distance $s$. The two coils carry equal currents $i$ in the same direction. (a) Show that the first derivative of the magnitude of the net magnetic field of the coils $(d B / d x)$ vanishes at the midpoint $P$ regardless of the value of $s$. Why would you expect this to be true from symmetry? (b) Show that the second derivative $\left(d^{2} B / d x^{2}\right)$ also vanishes at $P$, provided $s=R$. This accounts for the uniformity of $B$ near $P$ for this particular coil separation.
91 SSM A square loop of wire of edge length $a$ carries current $i$. Show that the magnitude of the magnetic field produced at a point on the central perpendicular axis of the loop and a distance $x$ from its center is

$$
B(x)=\frac{4 \mu_{0} i a^{2}}{\pi\left(4 x^{2}+a^{2}\right)\left(4 x^{2}+2 a^{2}\right)^{1 / 2}} .
$$

Prove that this result is consistent with the result shown in Problem 89.
92 Show that if the thickness of a toroid is much smaller than its radius of curvature (a very skinny toroid), then Eq. 29-24 for the field inside a toroid reduces to Eq. 29-23 for the field inside a solenoid. Explain why this result is to be expected.
93 SSM Show that a uniform magnetic field $\vec{B}$ cannot drop abruptly to zero (as is suggested by the lack of field lines to the right of point $a$ in Fig. 29-89) as one moves perpendicular to $\vec{B}$, say along the horizontal arrow in the figure. (Hint: Apply Ampere's law to the rectangular path shown by the dashed lines.) In actual magnets, "fringing" of the magnetic field lines always occurs, which means that $\vec{B}$ approaches zero in a gradual manner. Modify the field lines in the figure to indicate a more realistic situation.


Fig. 29-89 Problem 93.

